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INFLUENCE OF SUPPORTING CONDITIONS ON STRESS INTENSITY FACTORS FOR SINGLE-EDGE-CRACKED SPECIMENS UNDER BENDING

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16. Abstract

The stress intensity factors for single-edge-cracked (SEC) specimens under three- or four-point bending are analyzed by the body force doublet method and the influence of supporting conditions on K_1 is considered. The difference between the values of K_1 under pure bending and three-point bending is discussed, based on the difference between the stress distributions of the specimens without any crack under these two loading conditions. The approximate expression for K_1 of SEC specimens under three-point bending is also given for convenience of practical uses.

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INFLUENCE OF SUPPORTING CONDITIONS ON STRESS INTENSITY FACTORS FOR SINGLE-EDGE-CRACKED SPECIMENS UNDER BENDING

Hiromobu Nisitani* and Kazuya Mori**

1. Introduction

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Several analyses of the stress intensity factor K_1 on single edge cracked plate test specimens under bending, which were supported at three or four points (called SEC test specimens hereafter), were carried out in the past [1)-4)]. However, these studies were limited to the cases where the supporting points were fixed. Therefore, it is still unclear how changes of the supporting points will affect K_1.

Considering the above mentioned points, the authors computed highly accurate values of K_1 for various supporting conditions, by using the body force doublet method, which had been published before, by them. By considering the cases of the stress distributions where the specimen had no cracks, and effects of the side surfaces on cracks, and their interference, we investigated the physical background of effects of the supporting conditions. Further, we found an approximate equation for K_1 of SEG test specimens supported at three points, for the practical use. We also explained the limiting case where K_1 of SEC test specimens bent at three supporting points was approximated by the results of pure bending.

2. Theory of analysis

Figure 1 shows the problem to be analyzed, and the symbols. Especially, in the case of $d \to 0$ in the figure, the problem of three point bending was analyzed by superimposing the narrow plate under a periodical loading shown in Figure 2(a) and the plate under pure bending shown in Figure 2(b) and the narrow plate under a periodic body force doublet distribution. By

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these cases, we rigorously represented the singularity of the concentrated load in the four-point bending. The boundary conditions for shear stresses on the cracked surface and the side surface (hereafter, the straight line boundary perpendicular to the direction of the length is called the side surface) were automatically satisfied. By this approach, we improved the accuracy and reduced the number of unknown parameters. We concretely explain the method of analysis below.

2.1. Stress released by the body force doublet

As shown in Figure 2(a), we apply a periodic load on the straight edge of the narrow plate (we call the straight boundary parallel to the direction of the length, the straight edge, hereafter). In Figure 2(a), the stress in the narrow plate can be determined by the solution [5] of the case where a cosine wave stress is applied to the straight edge of the narrow plate, since the periodically acting concentrate force is expressed in a Fourier series. (The present problem does not involve the Poisson ratio V. Therefore, we assume V=0).

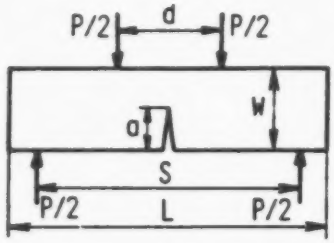

Figure 1 - Explanation of symbols

We denote the bending moment appearing in the side surface caused by the periodic loading by M_b (M_b is determined by numerical computation). We attempted to improve the accuracy by applying M_b, as shown in Figure 2(b), and thus making the net moment zero at this stage. /752

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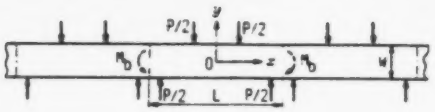

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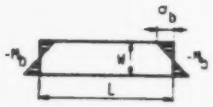

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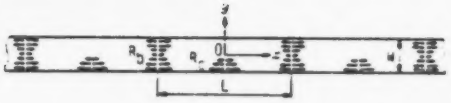

(c) -

Figure 2 - Periodic load applied to straight edge of narrow plate

2.2. Stress caused by the body force doublet

The stress field for the case of a narrow plate for which the periodical body force doublet is distributed on the cracked surface and the side surface can be calculated by using the stress field (Figure 3) of a narrow plate where a concentrated force doublet periodically acts [4].

We express the strengths of the body force doublets, acting on a small element $d\eta$ on the straight line, which is the boundary of the cracked and side surfaces by dR_a and dR_b. The densities of the individual body force doublets γ_c and γ_b in the present paper are defined by the following equations:

$$\left.\begin{aligned} \gamma_c &= \frac{1}{4\sqrt{a^2-(\eta+W/2)^2}}\frac{dR_a}{d\eta} \\ \gamma_b &= \frac{dR_b}{d\eta} \end{aligned}\right\} \qquad (1)$$

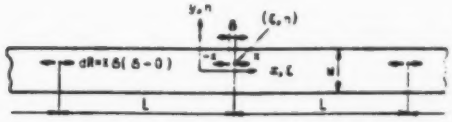

Figure 3 - The basic solution

2.3. Stress intensity factor

In order to determine γ_a and γ_b, we made a cut shown in Figure 4. (the number of divisions in the cracked surface is N_c and that for the side surface N_b), approximated γ_c and γ_b by step functions and determined γ_c and γ_b from the boundary conditions for the stresses at the middle point of each division.

The value of K_1 is obtained from the equation below.

$$K_1=\gamma_c|_{i=N_c}\sqrt{\pi a} \qquad (2)$$

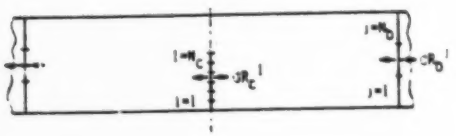

Figure 4. Division of the cracked surface and the side surface

3. Results of numerical calculation

We determined the stress intensity factors for SEC specimens bent by 3- and 4-point supportings by extrapolating the values found for N_c=14 and 18, while keeping the ratio N_c/N_b of the numbers of sections in the cracked and side surfaces constant.

3.1. SEC test speciment of three-point bending

Figure 5 shows K_1 for three-point bending speciments in a dimensionless form. In order to study effects of the interval of the supporting points S/W, we used K_1 for the case of pure bending ($S/W\to\infty$), which is equivalent in the bending moment to the three-point bending as the standard. In the three-point bending, the concentrated load lies on the extension of the crack. Therefore, even when the distance between the supporting points $S/W\tilde{=}10$, K_1 differs from that

of pure bending by more than 5% when S/W=0.2 and 0.4. The reason for this difference is that in a narrow plate specimen, without cracks, the stress distributions at the location where the cracks manifest, is supposed to differ in the three-point bending, and pure bending, as shown in Figure 6. Figure 6 shows the positions of the crack, and the stress distributions in the x-direction on the cross section AB for different support conditions. In the three-point bending, the stress distribution is shifted toward the compressed side from the edge A to the distance of $q \sim 0.4W$, compared to the case of pure bending. This works to reduce K_1. From the point to the edge B, the stress distribution shifts toward the stretched side, thus increasing K_1. Therefore, effects of the length of the crack on the ratio of K_1's for the three-point bending, and pure bending, is minimal at $a/W \tilde{=} 0.4$. When $a/W \tilde{=} 1$, K_1 is determined by the moment alone, and $F_1/F_1|_{S/W \to \infty}$ converges to unity. This corresponds to K_1 depending on the stress field and the crack length a in the range of $a/W \leq 0.1$, and in the range of $a/W \leq 0.9$, it depends on the moment of the ligament and the ligament length only (see Figure 7).

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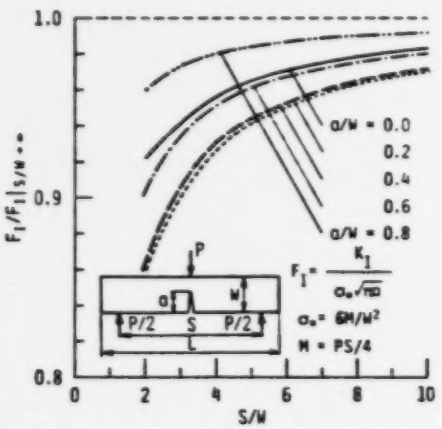

Figure 5. Effects of the distance between the supporting points on K_1 in the three-point bending

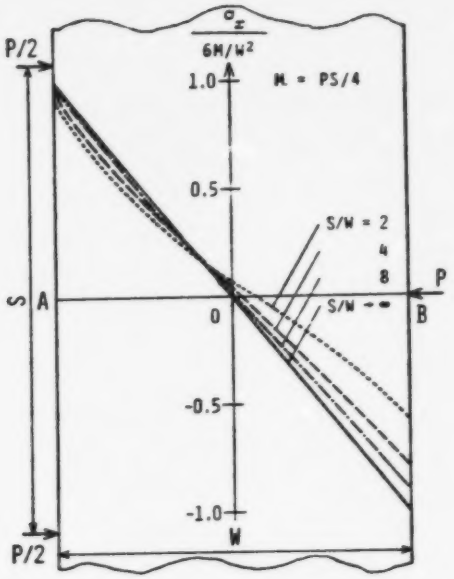

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Figure 6. The stress distribution in a narrow plate specimen without crack generated by the three-point bending

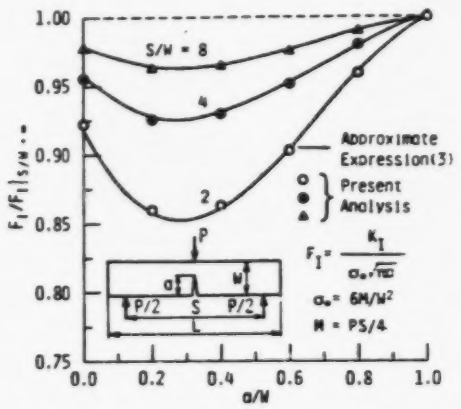

Figure 7. Effects of the crack length on K_1 in the three-point bending

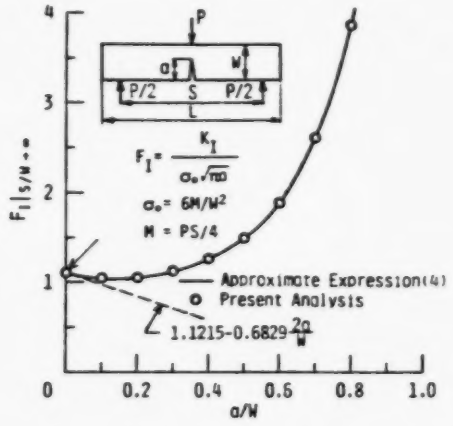

Figure 8. Effects of the crack length on K_1 in the pure bending ($S/W\to\infty$)

Next, for the purpose of practical application, we derive an approximate equation for K_1. In this derivation, we first express K_1 in the pure bending (see Figure 8) by Equation (4), and express K_1 for the threepoint bending by the ratio to the former.

$$\left.\begin{aligned} &K_1 = \sigma_0\sqrt{\pi a}\cdot F_1 \\ &F_1 = \beta\cdot F_1|_{S/W\to\infty} \quad \text{3 point bending} \\ &\sigma_0 = 3PS/2W^2 \\ &\xi = a/W \end{aligned}\right\} \quad (3)$$

$$F_1|_{S/W\to\infty} = 1.121 - 1.199\xi - 4.775\xi^2 - 1.628\xi^3 - 7.035\xi^4 + 13.27\xi^5 \quad \text{pure bending} \quad (4)$$

(for $\xi \leq 0.8$ error less than 0.5%)

$$\beta = 1 - 1.14(\xi - 1)(\xi + 0.12)(\xi - 1.18)/(S/W) \quad (5)$$

(for $S/W \geq 2$ error less than 0.5%)

3.2. SEC test specimen of the four-point bending

In this case, we fix the value of S-d, which directly relates to the bending moment, to S-d=2W, and describe effects of the supporting point gap d/W on K_1. Figure 9 shows K_1 for this case in a dimensionless form using the result of the pure bending ($d/W\to\infty$). As is seen from this figure, we can apply the approximation Equation (4) for the pure bending to finding K_1 of the four-point bending.

Next, as was done in section 3.1, the position of expected cracks in a narrow plate specimen without cracks in the four-point bending, and the stress distribution in the x-direction in the cross section AB, are shown in Figure 10 for the purpose of studying differences in K_1 for the four-point and pure bendings. The stress distribution for the four-point bending compared to that of the pure bending ($d/W\to\infty$), shows a trend similar to that of the three-point bending when $d/W<0.6$, and shifts towards the stretched side when $d/W>0.6$, which is contrary to the case of the three-point bending. Therefore, in Figure 9, the ratio of K_1's of the four-point and pure bendings exceeds unity when $d/W \geq 0.6$.

Figure 11 shows effects of the crack length on the ratio of K_1 of the four-point and pure bendings. In the region of $d/W \lesssim 0.6$, the trend is the same as in three-point bending.

3.3. Effects of side surface boundary

In the previous sections, we carried out our analysis by assuming the distance L between the side surfaces was sufficiently large so that its effect was negligible. In this section we study effects of L, or more directly, effects of L-S. Figure 12 shows effects of L in three-point bending where S/W is fixed at 4. From the figure, we see that the effects of the side surface boundary are negligible if $(1-S)/W > 0.1$.

Effects of the side surface boundary are:

(i) The stress field which is to be released by cracks

(ii) Interference effects between the crack as the side surface when an internal pressure is applied to the crack. In order to study effects of (ii) alone, we studied effects of L in pure bending. Results are shown in Figure 13. The Figure shows that the effect of (ii) is negligible if $L/W \geq 2$. In the case of Figure 12 where effects (i) and (ii) are simultaneously present, the effect of (i) is dominant.

From the above observation, effects of the side surface boundary seem to be negligible if $(L-S)/W \geq 0.1$ and $L/W \leq 2$.

4. Conclusion

We studied effects of the supporting conditions on the stress intensity factors for single-edge cracked specimens under bending by the body force doublet method, and obtained the results given below.

(1) In the three point bending, the concentrated load is on the extension of the crack, and its magnitude is inversely

proportional to the supporting point gap S/W, if the moment is kept constant. Therefore, the difference between K_1's for the three-point bending and the pure bending is characterized by an inverse proportionality to S/W. Consequently, the value of K_1 of the three-point bending does not converge quickly to that of the pure bending, and therefore, we can not approximate it by the pure bending.

(2) The value of K_1 of the three-point bending is determined by the released stress field and the crack length when $a/W \leq 0.1$. If $a/W \geq 0.9$, it depends only on the moment and the length of the ligament.

(3) In the four-point bending, effects of the supporting point gap d/W on K_1 are small, and if $d/W \geq 2$, it can be approximated by the pure bending.

(4) If the distance between the crack and the side surface boundary L/W is larger or equal to 2, and the distance between the supporting point and the side surface boundary $(L-S)/W \geq 0.1$, effects of the side surface boundary on K_1 is practically nil.

(5) We gave an approximate equation for obtaining accurate K_1 of SEC specimens under three-point bending, for practical use.

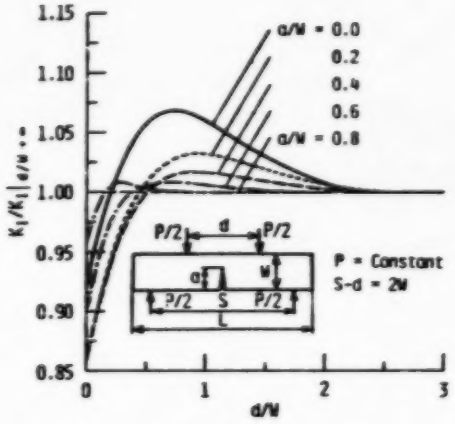

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Figure 9. Effects of the supporting point gap on K_1 under four-point bending (S-d=2W)

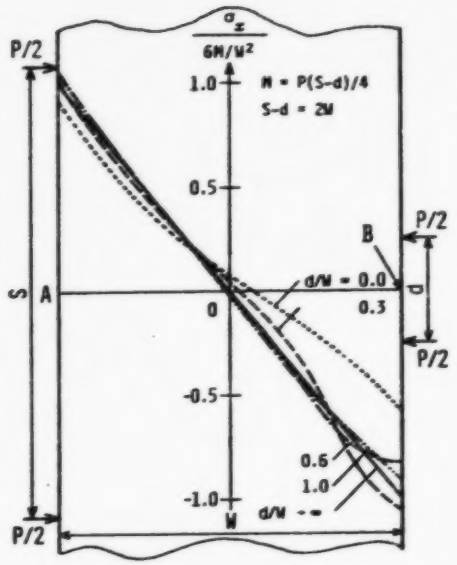

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Figure 10. The stress field in plate specimen without cracks under four-point bending (S-d=2W)

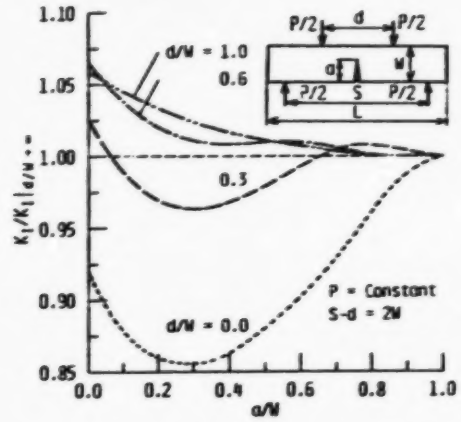

Figure 11. Effects of the crack length on K_1 under four-point bending

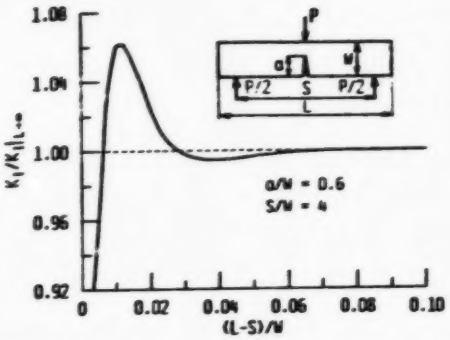

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Figure 12. Effects of the side surface boundary on K_1 under three-point bending

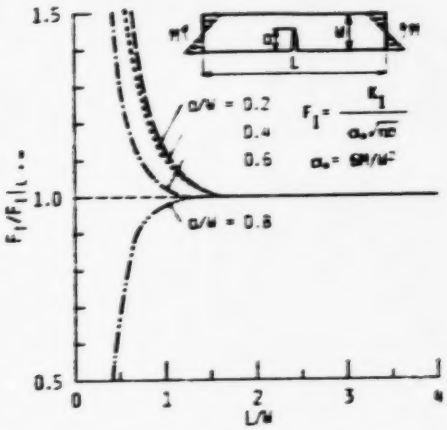

Figure 13. Effects of the side surface boundary on K_1 under pure bending

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